1. Consider the Gaussian probability density $P(x)$ for the continuous variable x,
$$P(x)dx = Ae^{-a(x-x_0)^2}dx$$
Determine the normalization constant A.

2. Using $P(x)$ from the previous problem, compute $\langle x \rangle$, $\langle x^2 \rangle$, and σ_x

3. Do the same operations as in problems 1 and 2, but for the Exponential distribution,
$$p(x)dx = ce^{-\lambda x}dx, \quad x \geq 0$$

4. According to the kinetic theory of gases, the energies of molecules moving along the x-direction are given by $\varepsilon_x = mv_x^2/2$ where m is the mass of the molecule and v_x is the velocity in the x-direction. The distribution of particles with a given velocity is given by the Boltzmann law,
$$p(v_x) = e^{-mv_x^2/2k_BT}$$
(This is sometimes called the Maxwell-Boltzmann distribution). Given that velocities can range from $-\infty$ to ∞,

   (a) Write the probability distribution $p(v_x)$ so that it is correctly normalized,
   (b) Compute the average energy, $\langle mv_x^2/2 \rangle$,
   (c) Find the average velocity $\langle v_x \rangle$, and
   (d) Find the average momentum $\langle mv_x \rangle$.

5. A biological membrane contains N ion-channel proteins. The fraction of time that any one protein is open to allow ions to flow through is q. Express the probability $P(m,N)$ that m of the channels will be open at any given time.

6. Problem 16–6 from McQuarrie & Simon:

   Research in surface science is carried out using ultra-high vacuum chambers that can sustain pressures as low as 10^{-12} torr. How many molecules are there in a 1.00-cm^3 volume inside such an apparatus at 298 K? What is the corresponding molar volume $\overline{V}$ at this pressure and temperature?

7. Problem 16–40 from McQuarrie & Simon:

   Using the Lennard-Jones parameters given in Table 16.7, compare the depth of a typical Lennard-Jones potential to the strength of a covalent bond.

8. 20 points extra credit: The Central Limit Theorem

   (a) Write a simple computer program (in whatever language you'd like) which generates the sum
   $$X_N = \sum_{i=1}^{N} \frac{x_i}{\sqrt{N}}$$
   where $\{x_1, ..., x_i, ...x_N\}$ are independent random numbers which are uniformly distributed on the interval $-1/2 < x_i < 1/2$. Your program should compute X_N at least one million times and then construct a histogram of the X_N values you observe.

(b) What are the maximum and minimum possible values for X_N?

(c) Show (numerically) that for large N the distribution of X_N values looks Gaussian.

(d) Where does the Gaussian approximation work best? Where does it fail?

Table 16.7: Lennard-Jones parameters, ε and σ, for various substances.

Species	$(\varepsilon/k_B)/\mathrm{K}$	σ/pm	$(2\pi\sigma^3 N_A/3)/\mathrm{cm}^3\cdot\mathrm{mol}^{-1}$
He	10.22	256	21.2
Ne	35.6	275	26.2
Ar	120	341	50.0
Kr	164	383	70.9
Xe	229	406	86.9
H_2	37.0	293	31.7
N_2	95.1	370	63.9
O_2	118	358	57.9
CO	100	376	67.0
CO_2	189	449	114.2
CF_4	152	470	131.0
CH_4	149	378	68.1
C_2H_4	199	452	116.5
C_2H_6	243	395	77.7
C_3H_8	242	564	226.3
$C(CH_3)_4$	232	744	519.4