1. Problem 17–2 from McQuarrie & Simon:

   Show that Equation 17.8 is equivalent to $f(x+y) = f(x)f(y)$. In this problem, we will prove that $f(x) \propto e^{ax}$. First, take the logarithm of the above equation to obtain

   $$\ln f(x+y) = \ln f(x) + \ln f(y)$$

   Differentiate both sides with respect to x (keeping y fixed) to get

   $$\left[\frac{\partial \ln f(x+y)}{\partial x}\right]_y = \frac{d \ln f(x+y)}{d(x+y)}\left[\frac{\partial (x+y)}{\partial x}\right]_y = \frac{d \ln f(x+y)}{d(x+y)}$$
   $$= \frac{d \ln f(x)}{dx}$$

   Now differentiate with respect to y (keeping x fixed) and show that

   $$\frac{d \ln f(x)}{dx} = \frac{d \ln f(y)}{dy}$$

   For this relation to be true for all x and y, each side must equal a constant, say a. Show that

   $$f(x) \propto e^{ax} \qquad \text{and} \qquad f(y) \propto e^{ay}$$

2. Problem 17–5 from McQuarrie & Simon:

   Show that the partition function in Example 17–1 can be written as

   $$Q(\beta, B_z) = 2\cosh\left(\frac{\beta\hbar\gamma B_z}{2}\right) = 2\cosh\left(\frac{\hbar\gamma B_z}{2k_B T}\right)$$

   Use the fact that $d\cosh x/dx = \sinh x$ to show that

   $$\langle E \rangle = -\frac{\hbar\gamma B_z}{2}\tanh\frac{\beta\hbar\gamma B_z}{2} = -\frac{\hbar\gamma B_z}{2}\tanh\frac{\hbar\gamma B_z}{2k_B T}$$

3. Problem 17–6 from McQuarrie & Simon:

   Use either the expression for $\langle E \rangle$ in Example 17–1 or the one in Problem 17–5 to show that

   $$\langle E \rangle \to -\frac{\hbar\gamma B_z}{2} \qquad \text{as} \qquad T \to 0$$

   and that

   $$\langle E \rangle \to 0 \qquad \text{as} \qquad T \to \infty$$

4. Problem 17–9 from McQuarrie & Simon:

   In Section 17–3, we derived an expression for $\langle E \rangle$ for a monatomic ideal gas by applying Equation 17.20 to $Q(N,V,T)$ given by Equation 17.22. Apply Equation 17.21 to

   $$Q(N,V,T) = \frac{1}{N!}\left(\frac{2\pi m k_B T}{h^2}\right)^{3N/2} V^N$$

   to derive the same result. Note that this expression for $Q(N,V,T)$ is simply Equation 17.22 with β replaced by $1/k_B T$.

5. Problem 17–11 from McQuarrie & Simon:

   Although we will not do so in this book, it is possible to derive the partition function for a monatomic van der Waals gas.

$$Q(N,V,T) = \frac{1}{N!}\left(\frac{2\pi m k_B T}{h^2}\right)^{3N/2} (V - Nb)^N \, e^{aN^2/Vk_BT}$$

   where a and b are the van der Waals constants. Derive an expression for the energy of a monatomic van der Waals gas.

6. Problem 17–20 from McQuarrie & Simon:

   Deriving the partition function for an Einstein crystal is not difficult (see Example 17–3). Each of the N atoms of the crystal is assumed to vibrate independently about its lattice position, so that the crystal is pictured as N independent harmonic oscillators, each vibrating in three directions. The partition function of a harmonic oscillator is

$$\begin{aligned} q_{\text{ho}}(T) &= \sum_{v=0}^{\infty} e^{-\beta\left(v+\frac{1}{2}\right)h\nu} \\ &= e^{-\beta h\nu/2} \sum_{v=0}^{\infty} e^{-\beta v h\nu} \end{aligned}$$

   This summation is easy to evaluate if you recognize it as the so-called geometric series (MathChapter I)

$$\sum_{v=0}^{\infty} x^v = \frac{1}{1-x}$$

   Show that

$$q_{\text{ho}}(T) = \frac{e^{-\beta h\nu/2}}{1 - e^{-\beta h\nu}}$$

   and that

$$Q = e^{-\beta U_0}\left(\frac{e^{-\beta h\nu/2}}{1 - e^{-\beta h\nu}}\right)^{3N}$$

   where U_0 simply represents the zero-of-energy, where all N atoms are infinitely separated.

Numbered equations:

$$f(E_1 - E_3) = f(E_1 - E_2)\, f(E_2 - E_3) \tag{17.8}$$

$$\langle E \rangle = -\left(\frac{\partial \ln Q}{\partial \beta}\right)_{N,V} \tag{17.20}$$

$$\langle E \rangle = k_B T^2 \left(\frac{\partial \ln Q}{\partial \beta}\right)_{N,V} \tag{17.21}$$

$$Q(N, V, \beta) = \frac{[q(V,\beta)]^N}{N!} \tag{17.22}$$

From Example 17–1:

$$\begin{aligned} Q(T, B_z) &= e^{\beta \hbar \gamma B_z/2} + e^{-\beta \hbar \gamma B_z/2} \\ &= e^{\hbar \gamma B_z/2k_B T} + e^{-\hbar \gamma B_z/2k_B T} \end{aligned}$$

$$\begin{aligned} \langle E \rangle &= -\left(\frac{\partial \ln Q}{\partial \beta}\right)_{B_z} = -\frac{1}{Q(\beta, B_z)}\left(\frac{\partial Q}{\partial \beta}\right)_{B_z} \\ &= -\frac{\hbar \gamma B_z}{2}\left(\frac{e^{\beta \hbar \gamma B_z/2} - e^{-\beta \hbar \gamma B_z/2}}{e^{\beta \hbar \gamma B_z/2} + e^{-\beta \hbar \gamma B_z/2}}\right) \\ &= -\frac{\hbar \gamma B_z}{2}\left(\frac{e^{\hbar \gamma B_z/2k_B T} - e^{-\hbar \gamma B_z/2k_B T}}{e^{\hbar \gamma B_z/2k_B T} + e^{-\hbar \gamma B_z/2k_B T}}\right) \end{aligned}$$